NASA TECHNICAL NOTE

NASA TN D-5896

C.1

NASA TN D-5896

METHOD FOR DETERMINATION OF THE ACCURACY OF CLOSED DISTANT STATIONARY ORBITS DETERMINED FROM RANGE RATE

by Alton P. Mayo

Langley Research Center
Hampton, Va. 23365

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1970

1. Report No. NASA TN D-5896	2. Government Accession No.	
4. Title and Subtitle METHOD FOR DETERMINATION OF THE ACCURACY OF CLOSED DISTANT STATIONARY ORBITS DETERMINED FROM RANGE RATE		5. Report Date July 1970
		6. Performing Organization Code
7. Author(s) Alton P. Mayo		8. Performing Organization Report No. L-7005
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23365		10. Work Unit No. 125-17-06-04
		11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note
		14. Sponsoring Agency Code

15. Supplementary Notes Part of the information presented herein was included in a thesis entitled "Accuracy of a Heliocentric Space Metric as Determined From Earth-Based Tracking" offered in partial fulfillment of the requirements for the degree of Doctor of Philosophy, Virginia Polytechnic Institute, Blacksburg, Virginia, December 1968.

16. Abstract

Orbit-determination-accuracy procedures were derived for inertially fixed orbits about distant bodies. This condition is realized only for binary stars but it is closely approximated over short time periods for orbits about the moon and the planets. Analysis of the results computed by using the stationary-orbit techniques would be indicative of the orbit-determination results computed by using short data arcs on slowly varying orbits.

Analytic expressions for uncertainty in the orbit parameters were derived by assuming that a measurement was made every degree of true anomaly. These expressions could be used for other measurement schedules by application of a correction given by the inverse square root of the number of observations. The equations, as presented, are fairly brief and are applicable to short-arc orbit-determination accuracy, linearity, and convergence studies.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Navigation Orbit determination Accuracy	Unclassified – Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	30	$3.00

*For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151

METHOD FOR DETERMINATION OF THE ACCURACY OF CLOSED DISTANT STATIONARY ORBITS DETERMINED FROM RANGE RATE*

By Alton P. Mayo
Langley Research Center

SUMMARY

Orbit-determination-accuracy procedures were derived for inertially fixed orbits about distant bodies. This condition is realized only for binary stars but it is closely approximated over short time periods for orbits about the moon and the planets. Analysis of the results computed by using the stationary-orbit techniques would be indicative of the orbit-determination results computed by using short data arcs on slowly varying orbits.

Analytic expressions for uncertainty in the orbit parameters were derived by assuming that a measurement was made every degree of true anomaly. These expressions could be used for other measurement schedules by application of a correction given by the inverse square root of the number of observations. The equations, as presented, are fairly brief and are applicable to short-arc orbit-determination accuracy, linearity, and convergence studies.

INTRODUCTION

Since a basic requirement for any space mission is that the orbit of the spacecraft be known, considerable attention has been given in the past decade to procedures for orbit determination. The basic data used in the procedures for computing the orbit are measurements made by an Earth based tracking system or measurements made by an onboard navigator. The computed estimates of the spacecraft position and velocity are always erroneous; the degree of error depends mainly on the type and accuracy of the measurements. It is standard practice to consider these factors in the orbit-determination process and to provide statistical data on the sizes of the errors that could possibly exist in the computed spacecraft position and velocity.

---

*Part of the information presented herein was included in a thesis entitled "Accuracy of a Heliocentric Space Metric as Determined From Earth-Based Tracking" offered in partial fulfillment of the requirements for the degree of Doctor of Philosophy, Virginia Polytechnic Institute, Blacksburg, Virginia, December 1968.

The accuracy of the prediction of future positions and velocities of the spacecraft and the accuracy to which the spacecraft can be controlled both depend on the orbit-determination accuracy. The planet's mass, the solar pressure, and relativistic effects, among other things, can be determined from measurements of the orbit. The accuracy of determination of these parameters is also basically related to the orbit-determination accuracy. Many studies have been made of the procedures for computing orbit-determination accuracy. Complete and lengthy numerical procedures have been developed for both interplanetary and orbital flight. Shorter analytic approximations have been developed for the interplanetary and planetary approach phases of the missions. (See refs. 1 and 2.)

Analytic solutions have the advantage of bringing out the parameters which affect the orbit-determination accuracy and contribute to an understanding of the problem which is more obscured in the lengthy numerical procedures. Analytic solutions are also simpler and more rapid to compute. In this paper, analytic approximations for the accuracy of orbit determination are developed for the case where the spacecraft is in a stationary closed orbit about a very distant planet and an Earth based tracker makes range rate measurements to the spacecraft. The results are derived from the classical binary-star orbit-determination technique which was developed decades ago in order to determine the orbits of binary stars from spectroscopic data.

The accuracy of orbit determination is directly influenced by the errors of the tracking system. In this paper random uncorrelated errors are considered to exist in the range rate measurements. The orbit-determination accuracies are obtained for two cases where (1) the range rate measurements are made at prespecified values of true anomaly and (2) the range rate measurements are made at prespecified values of time.

An example application of the procedures derived in the paper is given for an orbit about the planet Mars. The content of this paper is essentially an expansion of part of reference 3.

SYMBOLS

a	semimajor axis of orbit
$\tilde{a}$	orbit parameter, $na \sin i$
A_j,B_j	orbit parameters, where $j = 1,2,\ldots,6$ (see eqs. (12) to (16) and eq. (47))
$\hat{A}_5$	parameter in equation (42)

$[A]$	matrix of partial derivatives (see eq. (43))
$\bar{B}_5$	orbit parameter (see eq. (51))
$\hat{B}_5$	parameter in equation (39)
$[B]$	diagonal matrix of B_j parameters
$\left.\begin{matrix} C_{11}, C_{22}, \\ C_{33}, C_{44} \end{matrix}\right\}$	cofactors of inner matrix of equation (61)
$[\mathrm{cov}\ E]$	covariance matrix of orbit parameters
e	orbit eccentricity
E	orbit parameter (see eqs. (30) and (43))
$\vec{h}$	orbit-angular-momentum vector
i	orbit inclination
$I_1, I_2, \ldots, I_8$	integrals as defined in equations (52) to (59)
I_{jj}	diagonal elements of the information matrix
$[\mathrm{Info}]$	information matrix of orbit parameters
k_1	orbit parameter (see eq. (4))
K	number of orbits
n	mean anomaly rate
$\bar{Q}$	parameter in equation (36)
r	radius
t	time

t_p	time of pericenter passage
x,y,z	planet-centered rectangular coordinates
μ	gravitational constant
ν	true anomaly
ρ	range
$\dot{\rho}$	range rate
σ	standard deviation
$\sigma_{\dot{\rho}}$	standard deviation of range rate errors
$\overline{\psi}$	orbit parameter (see eq. (67))
ω	argument of periapsis
Ω	longitude of ascending node

Subscripts:

a	semimajor axis
$\tilde{a}$	orbit parameter, na sin i
e	eccentricity
j,k	indices
t_p	time of pericenter passage
ω	argument of periapsis

Matrix notation:

$\left\{ \right\}$	column matrix

$[\ ]$ rectangular or square matrix

$[\ ]^{-1}$ inverse of matrix $[\ ]$

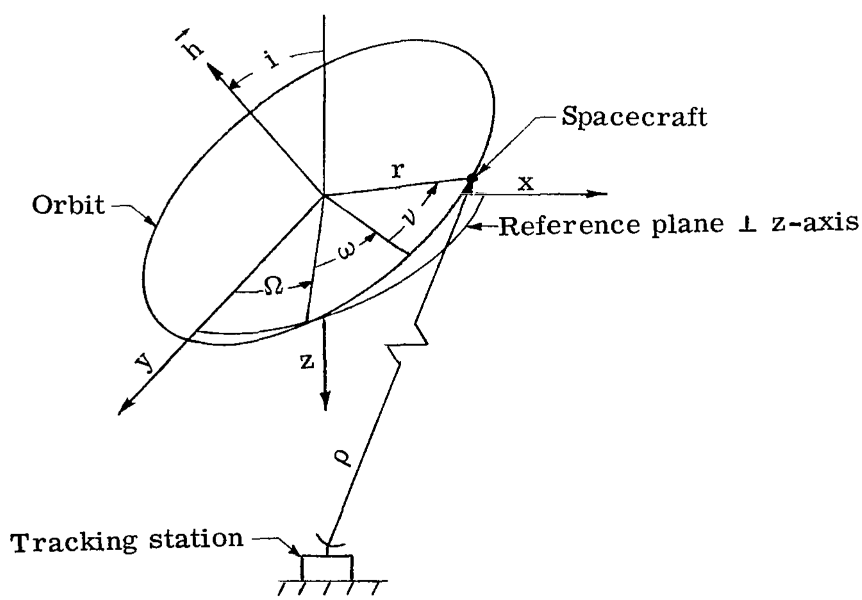
$[\ ]^{T}$ transpose of matrix $[\ ]$

A dot over a symbol denotes the derivative with respect to time.

PERTURBATION IN RANGE RATE CAUSED BY THE ORBIT-ELEMENT VARIATIONS

The orbit-determination-accuracy derivations of this paper are based on small-perturbation theory. In this section of the paper, approximate linear equations are derived which relate the small perturbations in range rate to small changes in the orbit parameters. In subsequent sections these small-perturbation equations are used in a weighted least-squares technique for calculating orbit-determination accuracy.

The study of range rate measurements of spacecraft motions in a distant stationary orbit has been the subject of many authors. (See refs. 4 and 5.) The range rate equations are generally developed by assuming that the effects of parallax may be neglected and by using a plane normal to the line of observation for referencing the orbit orientation. The choice of this particular reference plane simplifies the equations, since the range rate is insensitive to rotations of the ascending node of the orbit about the line of observation. The orbit elements are illustrated in the following sketch:



From observation of the sketch it can be deduced that the radar range to the spacecraft, with parallax neglected, is given by

$$\rho = r \sin i \sin(\nu + \omega) + z_{\text{tracking station}} \tag{1}$$

Taking the time derivative of equation (1), with i and ω assumed to be constants, gives

$$\dot{\rho} = \dot{r} \sin i \sin(\nu + \omega) + r \sin i \cos(\nu + \omega)\dot{\nu} + \dot{z}_{\text{tracking station}} \tag{2}$$

However, from the relations for a Keplerian orbit, $\dot{r}$ is expressed

$$\dot{r} = \frac{nae \sin \nu}{\sqrt{1 - e^2}} \qquad r\dot{\nu} = \frac{na(1 + e \cos \nu)}{\sqrt{1 - e^2}}$$

then it may by substitution be developed that

$$\dot{\rho} = \frac{na \sin i}{\sqrt{1 - e^2}}\left[e \cos \omega + \cos(\nu + \omega)\right] + \dot{z}_{\text{tracking station}} \tag{3}$$

By using the relation

$$n = \mu^{1/2} a^{-3/2}$$

and letting

$$k_1 = \frac{\mu^{1/2} a^{-1/2} \sin i}{\sqrt{1 - e^2}} \tag{4}$$

it follows that

$$\dot{\rho} = k_1\left[e \cos \omega + \cos(\nu + \omega)\right] + \dot{z}_{\text{tracking station}} \tag{5}$$

Expanding $\dot{\rho}$ in a Taylor's series and neglecting terms of higher order yield an expression for the perturbation in range rate due to small perturbations in the orbit elements, which is

$$\Delta\dot{\rho} = \frac{\partial\dot{\rho}}{\partial a} \Delta a + \frac{\partial\dot{\rho}}{\partial i} \Delta i + \frac{\partial\dot{\rho}}{\partial e} \Delta e + \frac{\partial\dot{\rho}}{\partial \omega} \Delta\omega + \frac{\partial\dot{\rho}}{\partial \nu} \Delta\nu \tag{6}$$

Since the component of the range rate due to the tracking-station motion is not a function of the orbit elements with parallax neglected the following partial derivatives are obtained:

$$\frac{\partial\dot{\rho}}{\partial a} = -\frac{k_1}{2a}\left[e \cos \omega + \cos(\nu + \omega)\right] \tag{7}$$

$$\frac{\partial\dot{\rho}}{\partial e} = \frac{k_1}{1 - e^2}\left[\cos \omega + e \cos(\nu + \omega)\right] \tag{8}$$

$$\frac{\partial\dot{\rho}}{\partial i} = k_1 \cot i\left[e \cos \omega + \cos(\nu + \omega)\right] \tag{9}$$

$$\frac{\partial\dot{\rho}}{\partial \omega} = k_1\left[-e \sin \omega - \sin(\nu + \omega)\right] \tag{10}$$

$$\frac{\partial\dot{\rho}}{\partial \nu} = k_1\left[-\sin(\nu + \omega)\right] \tag{11}$$

Also letting

$$A_1 = \frac{-k_1 e \cos \omega}{2a} \qquad B_1 = \frac{-k_1}{2a} \tag{12}$$

$$A_2 = \frac{k_1 \cos \omega}{1 - e^2} \qquad B_2 = \frac{k_1 e}{1 - e^2} \tag{13}$$

$$A_3 = k_1 e \cot i \cos \omega \qquad B_3 = k_1 \cot i \tag{14}$$

$$A_4 = -k_1 e \sin \omega \qquad B_4 = -k_1 \tag{15}$$

$$A_5 = 0 \qquad B_5 = -k_1 \tag{16}$$

equations (7) to (11) reduce to the form

$$\frac{\partial\dot{\rho}}{\partial a} = A_1 + B_1 \cos(\nu + \omega) \tag{17}$$

$$\frac{\partial\dot{\rho}}{\partial e} = A_2 + B_2 \cos(\nu + \omega) \tag{18}$$

$$\frac{\partial \dot{\rho}}{\partial i} = A_3 + B_3 \cos(\nu + \omega) \tag{19}$$

$$\frac{\partial \dot{\rho}}{\partial \omega} = A_4 + B_4 \sin(\nu + \omega) \tag{20}$$

$$\frac{\partial \dot{\rho}}{\partial \nu} = A_5 + B_5 \sin(\nu + \omega) \tag{21}$$

where the A_j and B_j coefficients are constants for a given orbit.

The small-perturbation equation (eq. (6)) is repeated as

$$\Delta\dot{\rho} = \frac{\partial \dot{\rho}}{\partial a}\Delta a + \frac{\partial \dot{\rho}}{\partial i}\Delta i + \frac{\partial \dot{\rho}}{\partial e}\Delta e + \frac{\partial \dot{\rho}}{\partial \omega}\Delta\omega + \frac{\partial \dot{\rho}}{\partial \nu}\Delta\nu$$

The perturbation in true anomaly $\Delta\nu$ may be transformed into a perturbation in time by the relation

$$\Delta\nu = \frac{\partial \nu}{\partial t}\Delta t$$

Thus

$$\Delta\dot{\rho} = \frac{\partial \dot{\rho}}{\partial a}\Delta a + \frac{\partial \dot{\rho}}{\partial i}\Delta i + \frac{\partial \dot{\rho}}{\partial e}\Delta e + \frac{\partial \dot{\rho}}{\partial \omega}\Delta\omega + \frac{\partial \dot{\rho}}{\partial \nu}\frac{\partial \nu}{\partial t}\Delta t \tag{22}$$

For the Keplerian elliptical orbit

$$\frac{\partial \nu}{\partial t} = \sqrt{\frac{\mu}{a^3\left(1 - e^2\right)^3}}(1 + e\cos\nu)^2$$

and

$$\frac{\partial \dot{\rho}}{\partial t} = \frac{\partial \dot{\rho}}{\partial \nu}\frac{\partial \nu}{\partial t} = \left[A_5 + B_5 \sin(\nu + \omega)\right]\sqrt{\frac{\mu}{a^3\left(1 - e^2\right)^3}}(1 + e\cos\nu)^2 \tag{23}$$

It is established in subsequent sections that, for a constant or zero error in the time of the measurement, an interrelation exists between the effects of inclination and semimajor-axis length which further reduces the number of parameters affecting the range rate to four.

It can be seen from equations (7) and (9) that

$$\frac{\partial \dot{\rho}}{\partial i} = -2a \cot i \frac{\partial \dot{\rho}}{\partial a} \tag{24}$$

Thus a small change Δa in the semimajor axis of the orbit produces the same perturbation in the range rate as a change in the orbit inclination equal to $-\frac{\Delta a}{2a \cot i}$. This makes it impossible to determine, in some actual situations, whether a perturbation in range rate is due to a change in the orbit inclination or a change in the semimajor axis.

Equation (22) now becomes

$$\Delta \dot{\rho} = \frac{\partial \dot{\rho}}{\partial a}(\Delta a - 2a \cot i \ \Delta i) + \frac{\partial \dot{\rho}}{\partial e} \Delta e + \frac{\partial \dot{\rho}}{\partial \omega} \Delta \omega + \frac{\partial \dot{\rho}}{\partial t_p} \Delta t_p \tag{25}$$

In equation (25) the perturbation of range rate is expressed in terms of the partial derivatives with respect to the orbit elements which are most commonly used in orbit-determination studies. However, the linear combination of parameters, $\Delta a - 2a \cot i \ \Delta i$, is difficult to visualize. In some cases, it may be easier to use an alternate parameter, na sin i, in the perturbation equations.

The first term on the right side of equation (25) can be expressed as the equality

$$\frac{\partial \dot{\rho}}{\partial a}(\Delta a - 2a \cot i \ \Delta i) = \frac{\partial \dot{\rho}}{\partial (na \sin i)} \frac{\partial (na \sin i)}{\partial a}(\Delta a - 2a \cot i \ \Delta i) \tag{26}$$

It can be deduced that

$$\frac{\partial (na \sin i)}{\partial i} = 2a \cot i \frac{\partial (na \sin i)}{\partial a} \tag{27}$$

thus

$$\Delta (na \sin i) = (\Delta a - 2a \cot i \ \Delta i) \frac{\partial (na \sin i)}{\partial a} \tag{28}$$

and combining equation (26) and equation (28) yields

$$\frac{\partial \dot{\rho}}{\partial a}(\Delta a - 2a \cot i \ \Delta i) = \frac{\partial \dot{\rho}}{\partial (na \sin i)} \Delta (na \sin i) \tag{29}$$

Because of the independence of the quantities na sin i, e, ω, and t_p equation (25), by use of equation (29), may be written in an alternate form as

$$\Delta \dot{\rho} = \frac{\partial \dot{\rho}}{\partial (na \sin i)} \Delta (na \sin i) + \frac{\partial \dot{\rho}}{\partial e} \Delta e + \frac{\partial \dot{\rho}}{\partial \omega} \Delta \omega + \frac{\partial \dot{\rho}}{\partial t_p} \Delta t_p \tag{30}$$

where from equation (3) it follows that

$$\frac{\partial \dot{\rho}}{\partial(\mathrm{na} \sin i)} = \frac{e \cos \omega}{\sqrt{1 - e^2}} + \frac{1}{\sqrt{1 - e^2}} \cos(\nu + \omega) \tag{31}$$

which is of the form

$$\frac{\partial \dot{\rho}}{\partial(\mathrm{na} \sin i)} = A_6 + B_6 \cos(\nu + \omega) \tag{32}$$

where $A_6 = \frac{e \cos \omega}{\sqrt{1 - e^2}}$ and $B_6 = \frac{1}{\sqrt{1 - e^2}}$.

Equation (30) applies to the case where the perturbation in true anomaly is due solely to the variation in the time of pericenter passage. In some cases, $\Delta\nu$ may be due to any of several factors. Consider the case where the range rate is sampled at prespecified times. The range rate measurements, sampled at prespecified times, correspond indirectly to measurements made at specified values of ν in the equation

$$t = \sqrt{\frac{a^3\left(1 - e^2\right)}{\mu}}\left[\frac{-e \sin \nu}{1 + e \cos \nu} + \frac{2}{\sqrt{1 - e^2}} \tan^{-1}\left(\frac{\sqrt{1 - e^2}}{1 + e} \tan \frac{\nu}{2}\right)\right] + t_p \tag{33}$$

For this case, the perturbation Δt is given by

$$\Delta t = \frac{\partial t}{\partial a} \Delta a + \frac{\partial t}{\partial e} \Delta e + \frac{\partial t}{\partial t_p} \Delta t_p$$

From equation (33)

$$\frac{\partial t}{\partial a} = \frac{3\left(t - t_p\right)}{2a} \tag{34}$$

and

$$\frac{\partial t}{\partial e} = -\frac{a^{3/2}}{\sqrt{\mu}} \frac{\sqrt{1 - e^2} \sin \nu}{1 + e \cos \nu} \overline{Q} \tag{35}$$

where

$$\overline{Q} = \frac{2.0 - e^2 - \frac{e(e + \cos \nu)}{1 + e \cos \nu}}{1 - e^2} \tag{36}$$

The corresponding perturbation equation (eq. (22)) for the range rate then takes the form

$$\Delta\dot{\rho} = \left(\frac{\partial\dot{\rho}}{\partial a} + \frac{\partial\dot{\rho}}{\partial\nu}\frac{\partial\nu}{\partial t}\frac{\partial t}{\partial a}\right)\Delta a + \left(\frac{\partial\dot{\rho}}{\partial e} + \frac{\partial\dot{\rho}}{\partial\nu}\frac{\partial\nu}{\partial t}\frac{\partial t}{\partial e}\right)\Delta e + \frac{\partial\dot{\rho}}{\partial i}\Delta i + \frac{\partial\dot{\rho}}{\partial\omega}\Delta\omega + \frac{\partial\dot{\rho}}{\partial\nu}\frac{\partial\nu}{\partial t}\Delta t_p \tag{37}$$

The first two parenthetical expressions each contain two terms. The first term represents the perturbations in range rate due to changes in the orbit parameters, if the perturbations in true anomaly are neglected. The second term is the effect of perturbations in true anomaly caused by changes in the orbit parameters. Let $\frac{\partial\tilde{\dot{\rho}}}{\partial a}$ represent the first parenthetical expression in equation (37), that is

$$\frac{\partial\tilde{\dot{\rho}}}{\partial a} = \frac{\partial\dot{\rho}}{\partial a} + \frac{\partial\dot{\rho}}{\partial\nu}\frac{\partial\nu}{\partial t}\frac{\partial t}{\partial a} \tag{38}$$

Then it follows from equations (17), (23), and (34) that

$$\frac{\partial\tilde{\dot{\rho}}}{\partial a} = A_1 + B_1 \cos(\nu + \omega) + \hat{B}_5(1 + e \cos \nu)^2 \sin(\nu + \omega)\left[\frac{3\left(t - t_p\right)}{2a}\right] \tag{39}$$

where

$$\hat{B}_5 = \frac{-\mu \sin i}{\left[a\left(1 - e^2\right)\right]^2}$$

or, in brief form, equation (39) becomes

$$\frac{\partial\tilde{\dot{\rho}}}{\partial a} = \hat{A} + \hat{B}\left[\frac{3\left(t - t_p\right)}{2a}\right] \tag{40}$$

The definitions of the periodic quantities $\hat{A}$ and $\hat{B}$ are inferred from equation (39).

Similarly, let $\frac{\partial\tilde{\dot{\rho}}}{\partial e}$ represent the second parenthetical expression in equation (37), that is

$$\frac{\partial\tilde{\dot{\rho}}}{\partial e} = \frac{\partial\dot{\rho}}{\partial e} + \frac{\partial\dot{\rho}}{\partial\nu}\frac{\partial\nu}{\partial t}\frac{\partial t}{\partial e} \tag{41}$$

and from equations (18), (23), and (35) it follows that

$$\frac{\partial\tilde{\dot{\rho}}}{\partial e} = A_2 + B_2 \cos(\nu + \omega) + \hat{A}_5 \sin(\nu + \omega)\sin \nu(1 + e \cos \nu)\overline{Q} \tag{42}$$

where $\overline{Q}$ is defined in equation (36) and

$$\hat{A}_5 = \frac{k_1}{1 - e^2}$$

In equations (35) and (42) the $\overline{Q}$ term is included whereas it was inadvertently omitted from equations (47b) and (50) in reference 3.

In equation (39) a secular term is now observed in the effects of the semimajor axis which did not exist for the case of zero or constant perturbation in the time. The perturbative effects of eccentricity are, however, nonsecular for both cases. The partial derivatives given by equations (19), (20), (23), (39), and (42) are shown in figures 1 to 5 for an orbit about Mars. (These figures are computer-generated plots.) The orbit elements for the Mars orbit are as follows:

a, km	5000.0
e	0.1000
i, radian	0.7000
ω, radian	0.6000
t_p, sec	0.0000

Figure 1 shows the secular and nonsecular components of $\frac{\partial \tilde{\dot{\rho}}}{\partial a}$. (See eq. (38).) The nonsecular component $\frac{\partial \dot{\rho}}{\partial a}$ is a simple cosine curve. This component gives the change in the range rate due to changes in the semimajor axis, with true anomaly held constant, and physically corresponds to range rate perturbations due to changes in a at prespecified values of true anomaly. The growth of the secular component $\frac{\partial \dot{\rho}}{\partial \nu} \frac{\partial \nu}{\partial t} \frac{\partial t}{\partial a}$ is caused by a change in the period of the orbit due to the change in the semimajor axis. More specifically, it is the result of measuring range rate on a time basis where there is a secular perturbation in the true anomaly of the spacecraft.

Two components of the partial derivative of range rate with respect to the orbit eccentricity $\frac{\partial \tilde{\dot{\rho}}}{\partial e}$ are shown in figure 2. The smaller component $\frac{\partial \dot{\rho}}{\partial e}$ expresses the range rate perturbations due to changes in eccentricity with true anomaly held constant. The larger component $\frac{\partial \dot{\rho}}{\partial \nu} \frac{\partial \nu}{\partial t} \frac{\partial t}{\partial e}$ is due to variations in true anomaly at the measurement sampling times as caused by a change in eccentricity. It can be deduced from figure 2 that the major effect of a change in eccentricity is the result of changes in the angular speed of the spacecraft. The changes are periodic and nonsecular.

The partial derivative of range rate with respect to the orbit inclination shown in figure 3 is a cosine curve of period equal to the orbit period and centered about the line $\frac{\partial \dot{\rho}}{\partial i} = A_3$. The cosine nature of the curve is evident from equation (19). The partial derivative $\frac{\partial \dot{\rho}}{\partial \omega}$ shown in figure 4 is a sine curve centered about the line $\frac{\partial \dot{\rho}}{\partial \omega} = A_4$ with a period equal to the orbit period. The partial derivative of the range rate with respect to time of pericenter passage is shown in figure 5. This curve is a plot of equation (23) and shows the effect of holding the orbit parameters fixed and shifting the true anomaly of the range rate measurement, at every point in orbit, corresponding to a unit increment in time. The partial derivative $\frac{\partial \dot{\rho}}{\partial t_p}$ is, as expected, periodic but not of simple shape.

ACCURACY OF THE ORBIT PARAMETERS WITH RANGE RATE MEASUREMENTS MADE EVERY DEGREE OF TRUE ANOMALY

The accuracy of the orbit parameters, as determined from tracking data, may be calculated by the method of weighted least squares. (See ref. 6.) In this method, the perturbation equations or equations of condition are obtained which correspond to each range rate measurement and can be written in the form

$$\left\{\Delta\dot{\rho}\right\} = \left[A\right]\left\{\Delta E\right\} \tag{43}$$

where the matrix $\left\{\Delta\dot{\rho}\right\}$ is a column matrix of measured differences between the range rate on the spacecraft's actual orbit and the corresponding values on the nominal orbit. The elements of matrix $\left[A\right]$ are partial derivatives of the range rate with respect to the orbit parameters, and the elements of the column matrix $\left\{\Delta E\right\}$ are increments in the orbit parameters. The information matrix, so called, is given by the equation

$$\left[\text{Info}\right] = \frac{1}{\sigma_{\dot{\rho}}^{2}}\left[A\right]^{T}\left[A\right]$$

where $\sigma_{\dot{\rho}}$ is the standard deviation of the errors in the range rate measurements, which are assumed to have a normal distribution and are uncorrelated. The general element of $\left[A\right]^{T}\left[A\right]$ is given by

$$\left(A^{T}A\right)_{jk} = \sum_{n} \frac{\partial\dot{\rho}}{\partial(\text{Parameter } j)} \frac{\partial\dot{\rho}}{\partial(\text{Parameter } k)} \tag{44}$$

If the observations are made every unit of true anomaly of the orbit, the approximation for a large number of measurements can be written

$$\left(A^T A\right)_{jk} \approx \int_{\nu_1}^{\nu_2} \frac{\partial \dot{\rho}}{\partial(\text{Parameter j})} \frac{\partial \dot{\rho}}{\partial(\text{Parameter k})} d\nu \tag{45}$$

From equation (45) the information matrix becomes

$$\frac{1}{\sigma_{\dot{\rho}}^2}\left[A\right]^T\left[A\right] \approx \frac{1}{\sigma_{\dot{\rho}}^2}\begin{bmatrix} \int_\nu \frac{\partial \dot{\rho}}{\partial \tilde{a}} \frac{\partial \dot{\rho}}{\partial \tilde{a}} d\nu & \int_\nu \frac{\partial \dot{\rho}}{\partial \tilde{a}} \frac{\partial \dot{\rho}}{\partial e} d\nu & \int_\nu \frac{\partial \dot{\rho}}{\partial \tilde{a}} \frac{\partial \dot{\rho}}{\partial \omega} d\nu & \int_\nu \frac{\partial \dot{\rho}}{\partial \tilde{a}} \frac{\partial \dot{\rho}}{\partial t_p} d\nu \\ & \int_\nu \frac{\partial \dot{\rho}}{\partial e} \frac{\partial \dot{\rho}}{\partial e} d\nu & \int_\nu \frac{\partial \dot{\rho}}{\partial e} \frac{\partial \dot{\rho}}{\partial \omega} d\nu & \int_\nu \frac{\partial \dot{\rho}}{\partial e} \frac{\partial \dot{\rho}}{\partial t_p} d\nu \\ & & \int_\nu \frac{\partial \dot{\rho}}{\partial \omega} \frac{\partial \dot{\rho}}{\partial \omega} d\nu & \int_\nu \frac{\partial \dot{\rho}}{\partial \omega} \frac{\partial \dot{\rho}}{\partial t_p} d\nu \\ \text{Symmetric} & & & \int_\nu \frac{\partial \dot{\rho}}{\partial t_p} \frac{\partial \dot{\rho}}{\partial t_p} d\nu \end{bmatrix} \tag{46}$$

where

$$\tilde{a} = na \sin i$$

In the following development previously derived analytic expressions (eqs. (18), (19), (20), (23), and (32)) are used for the partial derivatives in equation (46) and the integrations are performed to obtain analytic expressions for each element of the information matrix. The information matrix is then inverted to obtain analytic expressions for elements of the covariance matrix, that is

$$\left[\text{cov E}\right] = \left[\text{Info}\right]^{-1}$$

Only the diagonal elements of the covariance matrix will be obtained. These elements are the squares of the standard deviations of the errors in the orbit parameters. In deriving the expressions for the accuracy of the orbit parameters, two cases will be considered. The first case is for the range rate measurements made corresponding to prespecified values of true anomaly. The second case is where the measurements are made at prespecified values of time.

Range Rate Measurements Made at Prespecified Values of True Anomaly

When range rate measurements are made at prespecified values of true anomaly, the perturbation equation is equation (30) expressed as

$$\Delta\dot{\rho} = \frac{\partial\dot{\rho}}{\partial\tilde{a}}\Delta\tilde{a} + \frac{\partial\dot{\rho}}{\partial e}\Delta e + \frac{\partial\dot{\rho}}{\partial\omega}\Delta\omega + \frac{\partial\dot{\rho}}{\partial\nu}\frac{\partial\nu}{\partial t}\Delta t_p$$

where a perturbation exists in true anomaly which corresponds to a constant time increment Δt_p. This corresponds to an error in the location of pericenter-passage time in the true-anomaly time history. From equations (32), (18), (20), and (23), respectively,

$$\frac{\partial\dot{\rho}}{\partial\tilde{a}} = A_6 + B_6 \cos(\nu + \omega) \tag{47}$$

$$\frac{\partial\dot{\rho}}{\partial e} = A_2 + B_2 \cos(\nu + \omega) \tag{48}$$

$$\frac{\partial\dot{\rho}}{\partial\omega} = A_4 + B_4 \sin(\nu + \omega) \tag{49}$$

$$\frac{\partial\dot{\rho}}{\partial t_p} = \overline{B}_5(1 + e \cos \nu)^2 \sin(\nu + \omega) \tag{50}$$

where

$$\overline{B}_5 = B_5\sqrt{\frac{\mu}{a^3\left(1 - e^2\right)^3}} \tag{51}$$

In the formation of the information matrix the following integrals are needed:

$$I_1 = \int_0^{2K\pi} \cos(\nu + \omega)\, d\nu = 0 \tag{52}$$

$$I_2 = \int_0^{2K\pi} \sin(\nu + \omega)\, d\nu = 0 \tag{53}$$

$$I_3 = \int_0^{2K\pi} \sin^2(\nu + \omega)\, d\nu = K\pi \tag{54}$$

$$I_4 = \int_0^{2K\pi} \cos^2(\nu + \omega)\ d\nu = K\pi \tag{55}$$

$$I_5 = \int_0^{2K\pi} \sin(\nu + \omega)\ (1 + e \cos \nu)^2\ d\nu = 2K\pi e \sin \omega \tag{56}$$

$$I_6 = \int_0^{2K\pi} \sin^2(\nu + \omega)\ (1 + e \cos \nu)^2\ d\nu = \frac{K\pi}{4}\left(4 + e^2 + 2e^2 \sin^2\omega\right) \tag{57}$$

$$I_7 = \int_0^{2K\pi} \sin(\nu + \omega) \cos(\nu + \omega)\ (1 + e \cos \nu)^2\ d\nu = \frac{K\pi e^2}{2} \sin \omega \cos \omega \tag{58}$$

$$I_8 = \int_0^{2K\pi} \sin^2(\nu + \omega)\ (1 + e \cos \nu)^4\ d\nu = K\pi\left[1 + \frac{3e^2}{2} + \frac{e^4}{8} + \left(3e^2 - \frac{e^4}{8}\right)\sin^2\omega\right] \tag{59}$$

where K denotes the number of revolutions of the argument ν. Actually, K corresponds to the number of orbits over which the measurements are made. By substituting equations (47) to (50) into equation (46) and using equations (52) to (59), the information matrix becomes

$$\frac{1}{\sigma_{\dot{\rho}}^2}\left[A\right]^T\left[A\right] \approx \frac{1}{\sigma_{\dot{\rho}}^2}\begin{bmatrix} \left(2A_6^2 + B_6^2\right)K\pi & \left(2A_6A_2 + B_6B_2\right)K\pi & 2A_6A_4K\pi & A_6\overline{B}_5I_5 + B_6\overline{B}_5I_7 \\ & \left(2A_2^2 + B_2^2\right)K\pi & 2A_2A_4K\pi & A_2\overline{B}_5I_5 + B_2\overline{B}_5I_7 \\ & & \left(2A_4^2 + B_4^2\right)K\pi & A_4\overline{B}_5I_5 + B_4\overline{B}_5I_6 \\ \text{Symmetric} & & & \overline{B}_5^2I_8 \end{bmatrix} \tag{60}$$

However, from equations (13), (15), and (32)

$$A_2 = \frac{B_2 \cos \omega}{e}$$

$$A_4 = B_4 e \sin \omega$$

$$A_6 = B_6 e \cos \omega$$

and by substitution, equation (60) becomes

$$\frac{1}{\sigma_{\dot{\rho}}^2}[A]^T[A] \approx \frac{2K\pi}{\sigma_{\dot{\rho}}^2}[B]\begin{bmatrix} e^2\cos^2\omega + \frac{1}{2} & \cos^2\omega + \frac{1}{2} & e^2\sin\omega\cos\omega & \frac{5}{4}e^2\sin\omega\cos\omega \\ & e^{-2}\cos^2\omega + \frac{1}{2} & \sin\omega\cos\omega & \left(\frac{e^2}{4} + 1\right)\sin\omega\cos\omega \\ & & e^2\sin^2\omega + \frac{1}{2} & \frac{5}{4}e^2\sin^2\omega + \frac{(4 + e^2)}{8} \\ \text{Symmetric} & & & \frac{1}{16}\left[\left(8 + 12e^2 + e^4\right) + \left(24e^2 - e^4\right)\sin^2\omega\right] \end{bmatrix}[B] \tag{61}$$

where the matrix $[B]$ is a diagonal matrix of values B_1, B_2, B_4, and $\overline{B}_5$ or B_6, B_2, B_4, and $\overline{B}_5$, depending on whether the solution is for $\Delta a - 2a \cot i \, \Delta i$ or $\Delta(na \sin i)$. As previously mentioned, inverting the information matrix yields the covariance matrix, the diagonal elements of which are the squares of the standard deviations of the errors in the orbit parameters. In order to invert the information matrix of equation (61), only the inner matrix has to be inverted. The determinant of the inner matrix is given by

$$\det = \frac{\cos^2\omega}{64}\left[\frac{\left(e^6 + 14e^4 - 31e^2 + 16\right)}{2} - \left(5e^6 - 10e^4 + 5e^2\right)\sin^2\omega\right] \tag{62}$$

Note that the value of the determinant as well as the numeric condition of the information matrix is independent of the number of revolutions that the orbit is tracked, since K is not involved in the elements of the inner matrix.

The cofactors of the inner matrix are given by

$$\begin{aligned} C_{11} = &\left(\frac{e^2}{2}\sin^2\omega + \frac{e^{-2}}{2}\cos^2\omega + \frac{1}{4}\right)\frac{1}{16}\left[\left(8 + 12e^2 + e^4\right) + \left(24e^2 - e^4\right)\sin^2\omega\right] \\ &+ \left(\frac{e^2}{4} + 1\right)\sin^2\omega\cos^2\omega\left[\frac{5}{2}e^2\sin^2\omega + \frac{1}{2}\left(\frac{e^2}{4} + 1\right) - \left(\frac{e^2}{4} + 1\right)e^2\sin^2\omega\right] \\ &- \left(e^{-2}\cos^2\omega + \frac{1}{2}\right)\left(\frac{5}{4}e^2\sin^2\omega + \frac{4 + e^2}{8}\right)^2 \end{aligned} \tag{63}$$

$$C_{22} = \left(\frac{e^2}{2} + \frac{1}{4}\right)\frac{1}{16}\left[\left(8 + 12e^2 + e^4\right) + \left(24e^2 - e^4\right)\sin^2\omega + e^4\sin^2\omega\cos^2\omega\right]\left(\frac{25}{16}e^2\sin^2\omega\right.$$

$$\left. + \frac{5}{2}\,\frac{4 + e^2}{8} - \frac{25}{32}\right) - \left(e^2\cos^2\omega + \frac{1}{2}\right)\left(\frac{5}{4}e^2\sin^2\omega + \frac{4 + e^2}{8}\right)^2 \tag{64}$$

$$C_{33} = \frac{\cos^2\omega}{16}\left\{\left(\frac{e^2}{2} + \frac{e^{-2}}{2} - 1\right)\left[8 + 12e^2 + e^4 + \left(24e^2 - e^4\right)\sin^2\omega\right]\right.$$

$$+ \sin^2\omega\left[10\left(e^2 + 4\right)e^2\left(\cos^2\omega + \frac{1}{2}\right) - 25e^4\left(e^{-2}\cos^2\omega + \frac{1}{2}\right)\right.$$

$$\left.\left. - \left(e^2 + 4\right)^2\left(e^2\cos^2\omega + \frac{1}{2}\right)\right]\right\} \tag{65}$$

$$C_{44} = \left(e^{-2} + e^2 - 2\right)\frac{\cos^2\omega}{4} \tag{66}$$

By letting

$$\overline{\psi} = \left(\frac{2\pi K \times 57.295}{\sigma_{\dot{\rho}}^{2}/\det}\right)^{-1/2} \tag{67}$$

and using equations (63) to (66) the square roots of the diagonal elements of the covariance matrix – that is, the standard deviations of the errors in the orbit parameters – can be expressed as

$$\sigma_{\tilde{a}} \approx \overline{\psi} B_6^{-1}\sqrt{C_{11}} \tag{68}$$

$$\sigma_e \approx \overline{\psi} B_2^{-1}\sqrt{C_{22}} \tag{69}$$

$$\sigma_\omega \approx \overline{\psi} B_4^{-1}\sqrt{C_{33}} \tag{70}$$

$$\sigma_{t_p} \approx \overline{\psi} \overline{B}_5^{-1}\sqrt{C_{44}} \tag{71}$$

The limits on the integrals used to obtain equation (61) were expressed in radians. The constant 57.295 in equation (67) converts these limits to degrees since the summations being approximated (eq. (44)) are in degrees.

Equations (68) to (71) express the accuracies of the orbit parameters after tracking the spacecraft for K revolutions with a range rate measurement made every degree of true anomaly. A systematic error in the true anomaly, corresponding to a constant time increment, is considered to be solved for and it is assumed that no portion of the orbit is occulted. Equations (68) to (71) can be used for other measurement schedules by application of a correction proportional to the inverse square root of the number of observations. (See ref. 7.)

Considerable simplification of these expressions can be obtained by dropping the higher order terms in eccentricity for orbits where the eccentricity is much less than 1. Also, simplifications can be obtained for specific values of the argument of periapsis such as 0°, 45°, or 90°. For $\omega = 90^{\circ}$, however, the determinant of equation (62) becomes singular and the partial derivatives $\frac{\partial \dot{\rho}}{\partial e}$, $\frac{\partial \dot{\rho}}{\partial i}$, and $\frac{\partial \dot{\rho}}{\partial(\text{na sin i})}$ are linearly related. (See eqs. (8), (9), and (32).) The independent orbit parameters, in this case, become $\text{na sin i} + \frac{B_2}{B_6}\Delta e$, ω, and t_p.

It might be noted that some corrections have been made in equations (60) to (71) as compared to the corresponding equations in reference 3. However, the computed results of reference 3 correspond to the corrected equations of this paper.

For the case where there is no systematic error in the true anomaly, the perturbation equation becomes

$$\Delta\dot{\rho} = \frac{\partial \dot{\rho}}{\partial(\text{na sin i})}\Delta(\text{na sin i}) + \frac{\partial \dot{\rho}}{\partial e}\Delta e + \frac{\partial \dot{\rho}}{\partial \omega}\Delta\omega \tag{72}$$

and the information matrix (eq. (61)) becomes

$$\frac{1}{\sigma_{\dot{\rho}}^{2}}\left[A\right]^{T}\left[A\right] \approx \frac{2K\pi}{\sigma_{\dot{\rho}}^{2}}\left[B\right]\begin{bmatrix} e^{2}\cos^{2}\omega + \frac{1}{2} & \cos^{2}\omega + \frac{1}{2} & e^{2}\sin\omega\cos\omega \\ & e^{-2}\cos^{2}\omega + \frac{1}{2} & \sin\omega\cos\omega \\ & & e^{2}\sin^{2}\omega + \frac{1}{2} \\ \text{Symmetric} & & \end{bmatrix}\left[B\right] \tag{73}$$

In equation (73), only the effects of the range-rate-measurement error are considered in computing the information matrix. The effect on the range rate of a random error in the true anomaly of the measurement, if considered, could be approximated by changes in the value of $\sigma_{\dot{\rho}}$ to account for both error sources.

The uncertainties of the orbit parameters are obtained by inversion of the preceding information matrix (eq. (73)). By using the method of cofactors, the diagonal elements of the covariance matrix are obtained, and the square roots of the diagonal elements (which are the standard deviations of the orbit parameters) are thus given by

$$\sigma_{na\sin i} \approx \bar{\psi} B_6^{-1} \sqrt{\frac{e^{-2}\cos^2\omega + e^2\sin^2\omega + 1}{2}} \tag{74}$$

$$\sigma_e \approx \bar{\psi} B_2^{-1} \sqrt{\frac{2e^2 + 1}{4}} \tag{75}$$

$$\sigma_\omega \approx \bar{\psi} B_4^{-1} \sqrt{\cos^2\omega\left(\frac{e^2}{2} + \frac{e^{-2}}{2} - 1\right)} \tag{76}$$

where

$$\bar{\psi} = \left(\frac{2K\pi \times 57.295}{\sigma_{\dot{\rho}}^2/\det}\right)^{-1/2}$$

and

$$\det = \frac{\left(e^{-2} + e^2 - 2\right)\cos^2\omega}{4} \tag{77}$$

Range Rate Measurements Made at Prespecified Values of Time

In orbit-determination techniques, range rate measurements are usually made at prespecified values of time. For this case, the perturbation equation (eq. (37)) becomes

$$\Delta\dot{\rho} = \frac{\partial\tilde{\dot{\rho}}}{\partial a}\Delta a + \frac{\partial\tilde{\dot{\rho}}}{\partial e}\Delta e + \frac{\partial\dot{\rho}}{\partial i}\Delta i + \frac{\partial\dot{\rho}}{\partial\omega}\Delta\omega + \frac{\partial\dot{\rho}}{\partial\nu}\frac{\partial\nu}{\partial t}\Delta t_p \tag{78}$$

for which the diagonal elements I_{jj} of the information matrix become

$$I_{11} \approx \sigma_{\dot{\rho}}^{-2}\int_0^{2K\pi}\left(\frac{\partial\tilde{\dot{\rho}}}{\partial a}\right)^2 d\nu \tag{79}$$

$$I_{22} \approx \sigma_{\dot{\rho}}^{-2} \int_0^{2K\pi} \left(\frac{\partial \tilde{\dot{\rho}}}{\partial e}\right)^2 d\nu \tag{80}$$

$$I_{33} \approx \sigma_{\dot{\rho}}^{-2} \int_0^{2K\pi} \left(\frac{\partial \dot{\rho}}{\partial i}\right)^2 d\nu \tag{81}$$

$$I_{44} \approx \sigma_{\dot{\rho}}^{-2} \int_0^{2K\pi} \left(\frac{\partial \dot{\rho}}{\partial \omega}\right)^2 d\nu \tag{82}$$

$$I_{55} \approx \sigma_{\dot{\rho}}^{-2} \int_0^{2K\pi} \left(\frac{\partial \dot{\rho}}{\partial \nu}\right)^2 d\nu \tag{83}$$

By substituting equations (19), (20), (21), (39), and (42) in equations (79) to (83), the square roots of the diagonal elements of the covariance matrix are given in the following equations for the case where the off-diagonal elements of the information matrix are assumed to be zero:

$$\sigma_a \approx \left\{\sigma_{\dot{\rho}}^{-2} \sum_{\nu=1}^{360K} \left[A_1 + B_1 \cos(\nu + \omega) + \frac{3}{2a} \hat{B}_5 (1 + e \cos \nu)^2 \sin(\nu + \omega)\bar{q}_1\right]^2\right\}^{-1/2} \tag{84}$$

where

$$\bar{q}_1 = \sqrt{\frac{a^3(1 - e^2)}{\mu}} \left[\frac{-e \sin \nu}{1 + e \cos \nu} + \frac{2}{\sqrt{1 - e^2}} \tan^{-1}\left(\frac{\sqrt{1 - e^2}}{1 + e} \tan \frac{\nu}{2}\right)\right] + t_p$$

$$\sigma_e \approx \left\{K\sigma_{\dot{\rho}}^{-2} \sum_{\nu=1}^{360} \left[A_2 + B_2 \cos(\nu + \omega) + \hat{A}_5 \sin(\nu + \omega)\sin \nu(1 + e \cos \nu)\overline{Q}\right]^2\right\}^{-1/2} \tag{85}$$

where

$$\overline{Q} = \frac{2 - e^2}{1 - e^2} - \frac{e}{1 - e^2}\left(\frac{e \cos \nu}{1 + e \cos \nu}\right)$$

$$\sigma_i \approx \sigma_{\dot{\rho}}\left[2K\pi B_1^2\left(e^2\cos^2\omega + \frac{1}{2}\right) \times 57.295\right]^{-1/2} \left(2a \cot i\right)^{-1} \tag{86}$$

$$\sigma_{\omega} \approx \sigma_{\dot{\rho}}\left[2K\pi B_4{}^2\left(e^2\sin^2\omega + \frac{1}{2}\right) \times 57.295\right]^{-1/2} \tag{87}$$

$$\sigma_{t_p} \approx 4\sigma_{\dot{\rho}}\left\{2K\pi\overline{B}_5{}^2\left[\left(8 + 12e^2 + e^4\right) + \left(24e^2 - e^4\right)\sin^2\omega\right] \times 57.295\right\}^{-1/2} \tag{88}$$

It might be mentioned that for range rate measured at prespecified values of time five of the orbit elements are determinable, whereas for range rate measured at prespecified values of true anomaly only four orbit parameters could be determined. For the range rate measurements made at prespecified values of time, the partial derivatives and the information matrix obtained through the use of equations (19), (20), (23), (39), (42), and (44) are compared in table I with the secant-method results, which were obtained by finite differencing equations (3) and (33), and using equation (23), to obtain the partial derivatives (eq. (37)) that were applied in equation (44). The necessary relation between ν and t was obtained from Kepler's equation.

SUMMARY OF RESULTS

Equations for calculating orbit-determination accuracy were derived for inertially fixed orbits about distant bodies. This condition is realized only for binary stars but is closely approximated over short time periods for orbits about the Moon and the planets. Analysis of the results computed by using the stationary-orbit techniques would be indicative of the orbit-determination results computed by using short data arcs on slowly varying orbits.

Analytic expressions for uncertainty in the orbit elements were derived by assuming that a measurement was made every degree of true anomaly. These expressions could be used for many other measurement schedules by application of a correction given by the inverse square root of the number of observations.

Secular terms appeared in the partial derivative of range rate with respect to the semimajor axis $\frac{\partial\tilde{\dot{\rho}}}{\partial a}$ when measurements were considered to be made at prespecified values of time; however, the other partial derivatives remained periodic. The secular component of $\frac{\partial\tilde{\dot{\rho}}}{\partial a}$ was shown to be much larger than the nonsecular component by the end of the first revolution of the orbit. The main effect of measuring range rate at prespecified values of time is an improvement in the accuracy of determination of the semimajor axis and the eccentricity. This is largely caused by the variations in true anomaly of the spacecraft at given times with changes in semimajor axis and eccentricity. If measurements are sampled at prespecified values of true anomaly, the numeric condition of the information matrix does not vary with the number of complete orbits tracked; however, if the range rate measurements are made at prespecified values of time, the numeric condition is affected by the number of orbits tracked. It was shown that only three of the orbit parameters can be determined if the measurements are made with no systematic error in the prespecified values of true anomaly. For this case, all partial derivatives and residuals in the differential correction process are composed of sine and cosine curves of period of the orbit and the process is linear. However, it is not likely that measurements will be made at prespecified values of true anomaly on a distant orbit. Making measurements at prespecified values of time is operationally feasible and also permits determination of five of the orbit elements.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., May 7, 1970.

REFERENCES

1. Hamilton, T. W.; and Melbourne, W. G.: Information Content of a Single Pass of Doppler Data From a Distant Spacecraft. The Deep Space Network, Space Programs Sum. No. 37-39, Vol. III (Contract No. NAS 7-100), Jet Propulsion Lab., California Inst. Technol., May 31, 1966, pp. 18-23.

2. Nishimura, T.: Effect of Constant Acceleration on the Information Content of a Single Pass of Doppler Data. The Deep Space Network, Space Programs Sum. 37-42, Vol. III (Contract no. NAS 7-100), Jet Propulsion Lab., California Inst. Technol., Nov. 30, 1966, pp. 3-7.

3. Mayo, Alton P.: Accuracy of a Heliocentric Space Metric as Determined From Earth-Based Tracking. Ph. D. Thesis, Virginia Polytech. Inst., 1968.

4. Smart, W. M.: Text-Book on Spherical Astronomy. Fourth ed., Cambridge Univ. Press, 1944, pp. 340-367. (Reprinted 1956.)

5. Deutsch, Armin J.: Orbits for Planetary Satellites From Doppler Data Alone. ARS J., vol. 30, no. 6, June 1960, pp. 536-542.

6. Cramér, Harald: Mathematical Methods of Statistics. Princeton Univ. Press, 1951.

7. Mayo, Alton P.; and Adams, William M., Jr.: Analytic Approximations for Application to Spacecraft Optical Navigation on Short Data Arcs. NASA TN D-4863, 1968.

TABLE I.- COMPARISON OF ANALYTIC AND SECANT-METHOD PARTIAL DERIVATIVES AND INFORMATION MATRIX

[Measurements made at prespecified times]

(a) Partial derivatives at 270° true anomaly

$\partial\dot{\rho}/\partial a$	$\partial\dot{\rho}/\partial e$	$\partial\dot{\rho}/\partial i$	$\partial\dot{\rho}/\partial\omega$	$\partial\dot{\rho}/\partial t_p$
		Analytic method		
-2.46661707E-03	-1.47424969E+00	1.45852337E+00	1.45950649E+00	-9.32603735E-04
		Secant method		
-2.46654641E-03	-1.47413033E+00	1.45846323E+00	1.45944432E+00	-9.32571518E-04

(b) Information matrix (measurement every degree for 360°)

Analytic method:

$$\begin{bmatrix} 4.16525965E{+}06 & 1.21396297E{+}09 & -1.16732222E{+}09 & -3.35333990E{+}09 & 1.99564975E{+}06 \\ 1.21396297E{+}09 & 4.61695670E{+}12 & 4.65639996E{+}11 & 1.20256341E{+}11 & -7.20949554E{+}07 \\ -1.16732222E{+}09 & 4.65639996E{+}11 & 6.40560986E{+}12 & -3.33031794E{+}10 & 2.62833005E{+}07 \\ -3.35333990E{+}09 & 1.20256341E{+}11 & -3.33031794E{+}10 & 4.52367742E{+}12 & -2.70325479E{+}09 \\ 1.99564975E{+}06 & -7.20949554E{+}07 & 2.62833005E{+}07 & -2.70325479E{+}09 & 1.63135912E{+}06 \end{bmatrix}$$

Secant method:

$$\begin{bmatrix} 4.19999914E{+}06 & 1.24167263E{+}09 & -1.13480466E{+}09 & -3.37119924E{+}09 & 2.00731728E{+}06 \\ 1.24167263E{+}09 & 4.63899313E{+}12 & 4.91269987E{+}11 & 1.05961572E{+}11 & -6.27601418E{+}07 \\ -1.13480466E{+}09 & 4.91269987E{+}11 & 6.48476644E{+}12 & -5.01100596E{+}10 & 3.72468989E{+}07 \\ -3.37119924E{+}09 & 1.05961572E{+}11 & -5.01100596E{+}10 & 4.53288956E{+}12 & -2.70927273E{+}09 \\ 2.00731723E{+}06 & -6.27601418E{+}07 & 3.72468989E{+}07 & -2.70927273E{+}09 & 1.63529086E{+}06 \end{bmatrix}$$

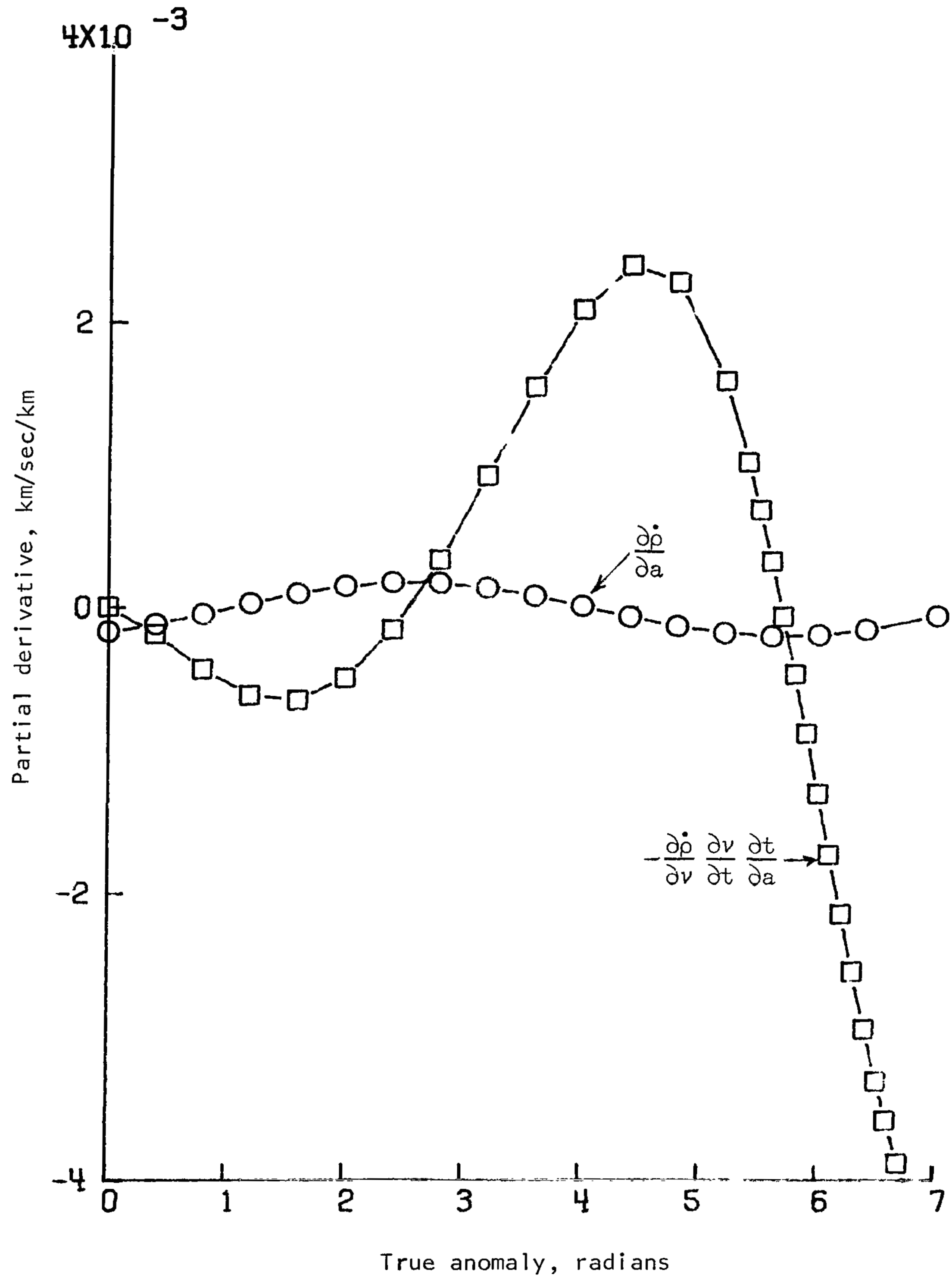


Figure 1.- Partial derivative of range rate with respect to changes in semimajor axis.

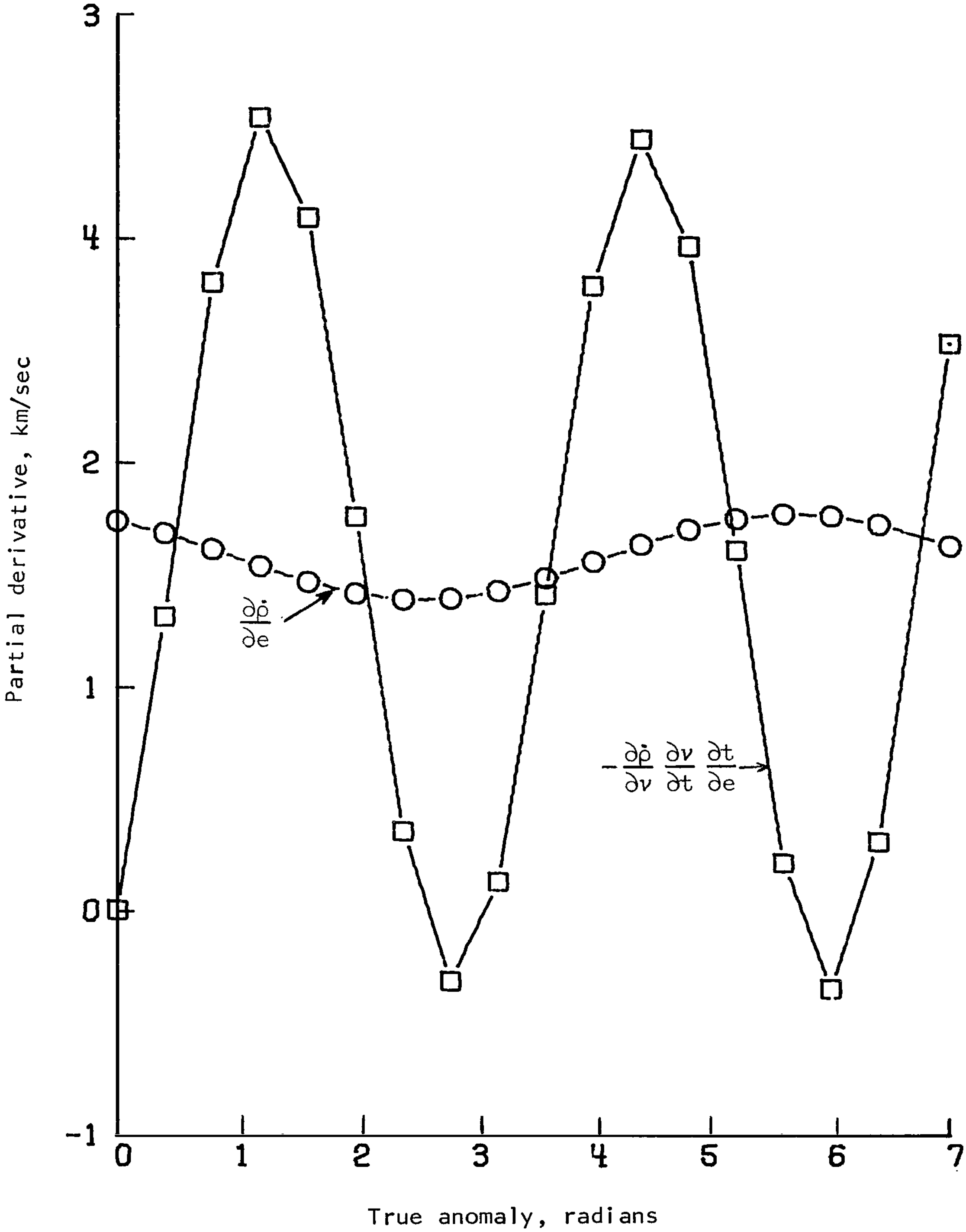


Figure 2.- Partial derivative of range rate with respect to changes in eccentricity.

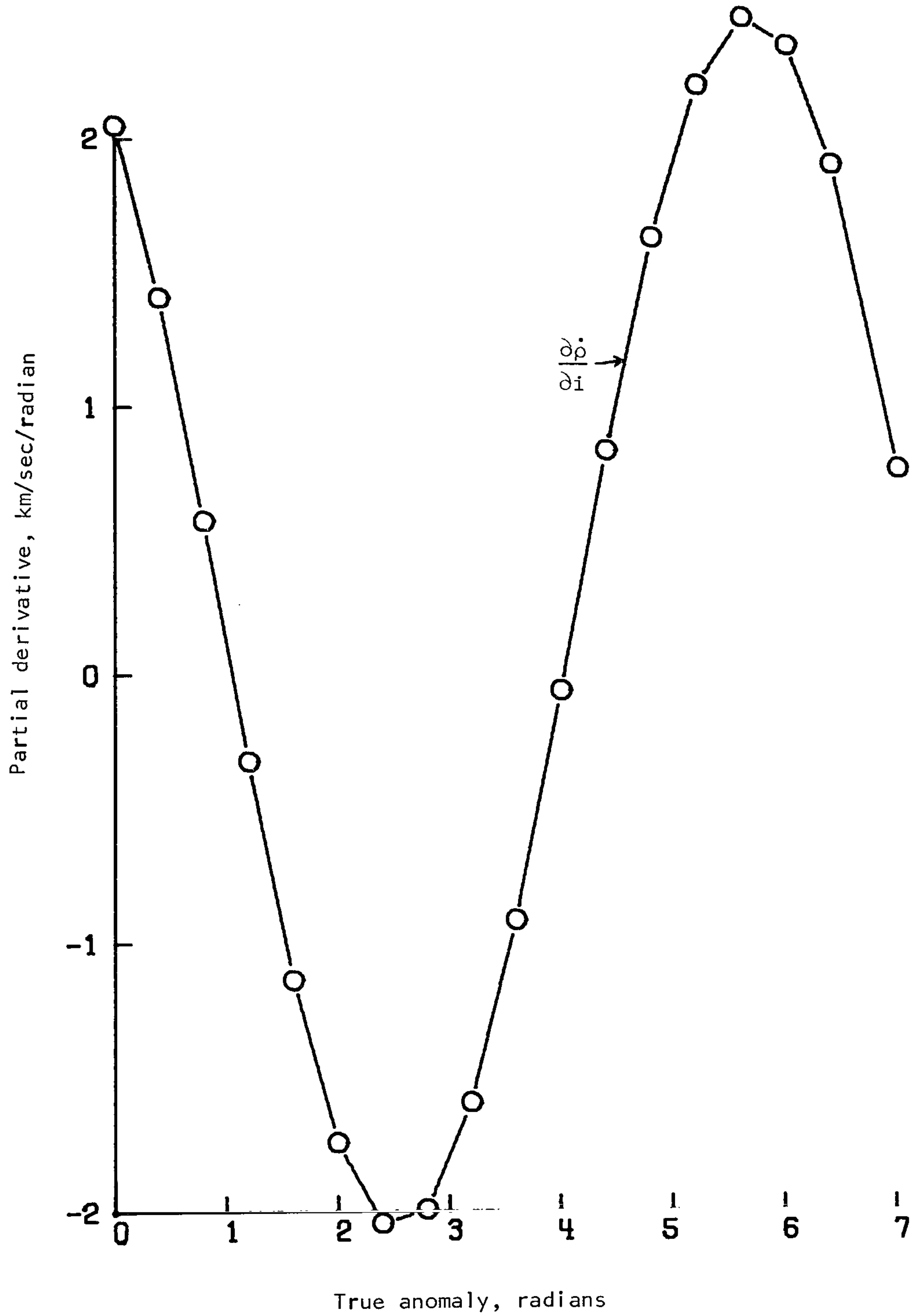


Figure 3.- Partial derivative of range rate with respect to changes in inclination.

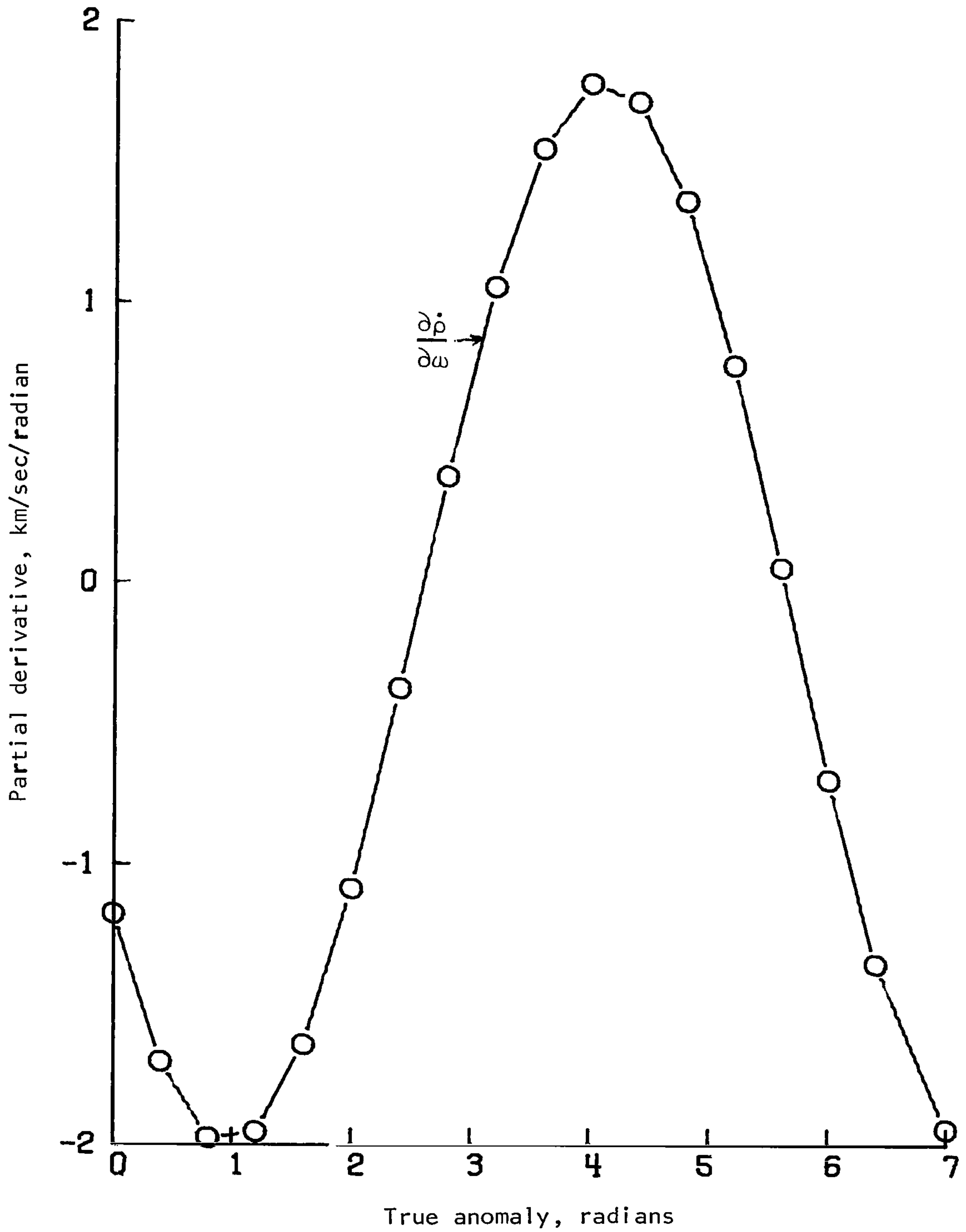


Figure 4.- Partial derivative of range rate with respect to changes in argument of pericenter.

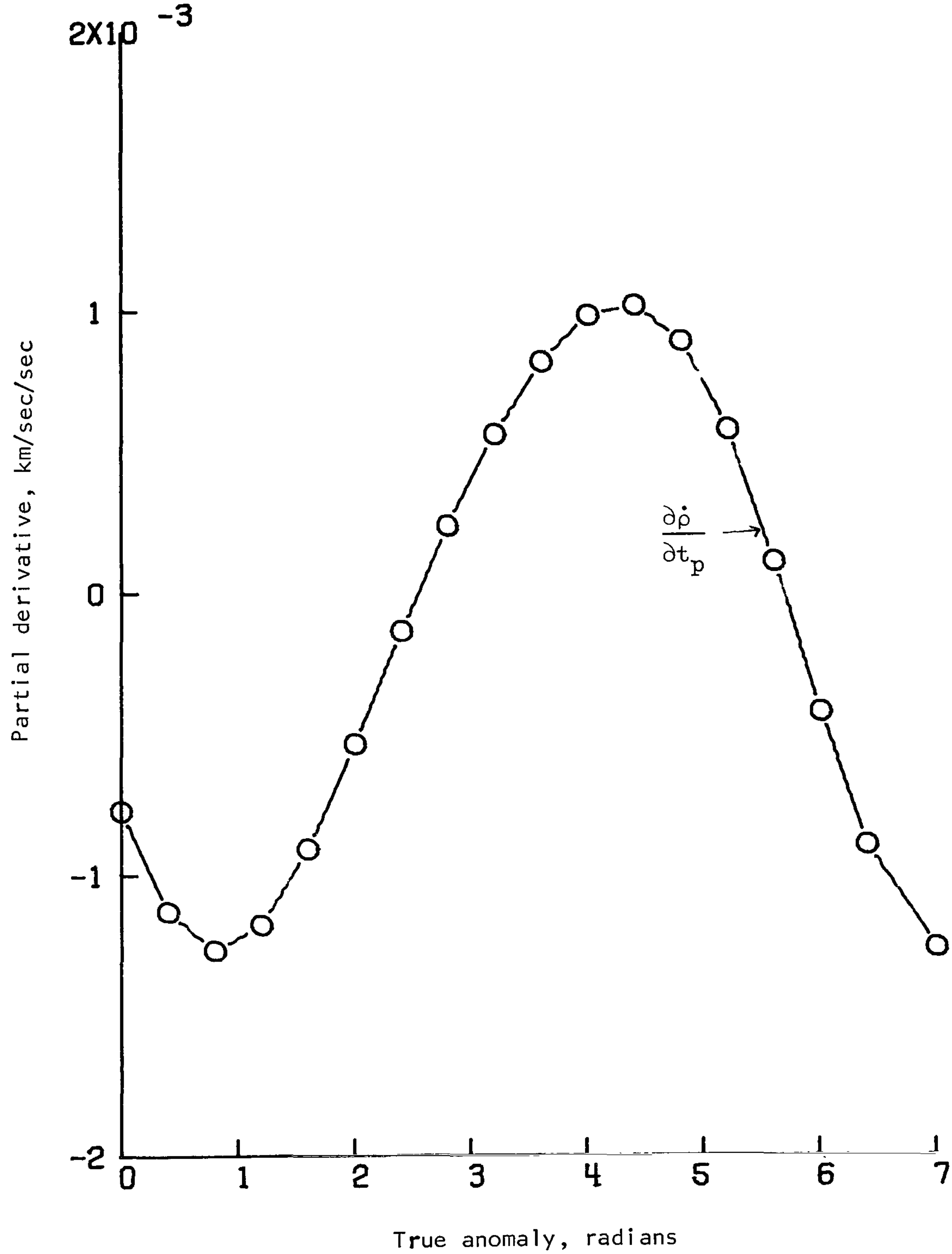


Figure 5.- Partial derivative of range rate with respect to changes in time of pericenter passage.

NASA-Langley, 1970 —— 30 L-7005